Calculus Formulas

Limits & Derivatives

Limit Notation

$$\lim_{x \to a} f(x) = L$$

"the limit of f(x) as x approaches a equals L"

$$\lim_{x \to a^-} f(x) = L$$

"the limit of f(x) as x approaches a <u>from the left</u> equals L"

$$\lim_{x \to a^+} f(x) = L$$

"the limit of f(x) as x approaches a <u>from the right</u> equals L"

Common Limits

$$\lim_{x \to 0^+} ln\, x = -\infty$$

$$\lim_{x \to \infty} \frac{1}{x^r} = 0 \; for \; r > 0$$

$$\lim_{x \to -\infty} e^x = 0$$

Testing Limits Left and Right

$\lim_{x \to a} f(x) = L$ if and only if

$$\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$$

Squeeze Theorem

If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then,

$$\lim_{x \to a} g(x) = L$$

Limit Definitions of a Derivative

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Derivative as a function

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Rules for Derivatives

$\frac{d}{dx}(c) = 0$ for constant c

$$\frac{d}{dx}(x) = 1$$

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$\frac{d}{dx}(f(x) * g(x)) = f(x) * \frac{d}{dx}(g(x)) + g(x) * \frac{d}{dx}(f(x))$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) * \frac{d}{dx}(f(x)) - f(x) * \frac{d}{dx}(g(x))}{[g(x)]^2}$$

$$[f(g(x))]' = f'(g(x)) * g'(x)$$

Fundamental Derivatives

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(a^x) = a^x ln(a)$$

$$\frac{d}{dx}(lnx) = \frac{1}{x}$$

$$\frac{d}{dx}(log_a x) = \frac{1}{x ln(a)}$$

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Trig Derivatives

$\frac{d}{dx}(sin(x)) = cos(x)$

$\frac{d}{dx}(cos(x)) = -sin(x)$

$\frac{d}{dx}(tan(x)) = sec^2(x)$

$\frac{d}{dx}(csc(x)) = -csc(x)cot(x)$

$\frac{d}{dx}(sec(x)) = sec(x)tan(x)$

$\frac{d}{dx}(cot(x)) = -csc(x)cot(x)$

Inverse Trig Derivatives

$\frac{d}{dx}(cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(tan^{-1}x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(csc^{-1}x) = -\frac{1}{x\sqrt{1-x^2}}$

$\frac{d}{dx}(sec^{-1}x) = \frac{1}{x\sqrt{1-x^2}}$

$\frac{d}{dx}(cot^{-1}x) = -\frac{1}{1+x^2}$

$\frac{d}{dx}(sin^{-1}x) = \frac{1}{\sqrt{1-x^2}}$

Hyperbolic Trig Derivatives

$\frac{d}{dx}(sinh(x)) = cosh(x)$

$\frac{d}{dx}(cosh(x)) = sinh(x)$

$\frac{d}{dx}(tanh(x)) = sech^2(x)$

$\frac{d}{dx}(csch(x)) = -cosh(x)coth(x)$

$\frac{d}{dx}(sech(x)) = -sech(x)tanh(x)$

$\frac{d}{dx}(coth(x)) = -csch^2(x)$

L'hospital's Rule

For indeterminate form of type $\frac{0}{0}$ or $\frac{\infty}{\infty}$,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$